Midterm exam Calculus-3 (10 points for free)

Problem 1
(10 points)

Suppose that $\Sigma_{n=1}^{\infty} a_n$ $(a_n \neq 0)$ is known to be a convergent series. Prove that $\Sigma_{n=1}^{\infty} 1/a_n$ is a divergent series.

Problem 2
(15 points)

(a, 5 points) Calculate the limit $\lim_{n\to\infty}\left\{\left(\sin\frac{1}{n^3}\right)/\left(\frac{1}{n^3}\right)\right\} = ?$

(b, 10 points) Is the series $\sum_{n=1}^{+\infty} \sin\frac{1}{n^3}$ convergent?

Problem 3
(15 points)

Consider the series $\sum_{n=1}^{\infty} \frac{(5x-4)^n}{n^3}$

For which values of x is the series convergent ?

Problem 4
(10 points)

(a: 10 points) Find the value of c=c(D) for which $\sum_{n=0}^{+\infty} e^{nc} = D\,(>1)$

Problem 5
(10 points)

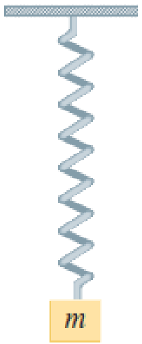
For the function $f_n(x) = \dfrac{4}{(x^2+1)n^6}$ with $x \in (-\infty, +\infty)$

(a: 3 points) Determine the pointwise limit $\lim_{n\to+\infty} f_n(x) = ?$

(b: 7 points) check if $f_n(x)$ is uniform convergent

Problem 6
(30 points)

Suppose a spring has mass m and spring constant k and let $\omega = \sqrt{k/m}$. Suppose that the damping constant is so small that the damping force is negligible. If an external force $F(t) = F_0 \cos \omega_0 t$ is applied, where $\omega_0 \neq \omega$,



Equation of motion: $$m\frac{d^2x}{dt^2} + c\frac{dx}{dt} + kx = F(t)$$

Show that in absence of damping (**c=0**) the motion is described by:

$$x(t) = c_1 \cos \omega t + c_2 \sin \omega t + \frac{F_0}{m(\omega^2 - \omega_0^2)} \cos \omega_0 t$$